With exponential functions, it is possible to have many different equations that represent a single curve on a graph. The reason: we can change the base of a function and adjust the coefficient in the exponent in such a way that nothing changes on the graph. The following two equations have coincident curves (check with DESMOS):

$$y = 150\ (6.25)^{3x}$$

$$y = 150\ (0.4)^{-6x}$$

Note: No other quantities in an exponential equation can be changed in a similar way; vertical stretches, vertical reflections and both translations have to be the same in equivalent functions.

Usually we are concerned with only two types of equivalent exponential functions: two equations for general growth rate and doubling time (base 2); or two equations for general decay rate and half-life (base $\frac{1}{2}$).

Two examples will show how to change base in an exponential equation and involve the equations in the document `Gr11_ExponentialsEquationsTable.pdf`.

1. A small town has a population of 500 in the year 2000. It grows at a rate of 5.5% per year. How long does it take for the population to double (find a time that gives an answer of 2.00, accurate to the nearest 100th).

   Use the general growth rate equation with $r = 0.055$:

   $$P(t) = 500\ (1.055)^t$$

   It is required to find t when $P(t) = 1000$. After dividing both sides by 500:

   $$2 = (1.055)^t$$

   The algebraic way to solve an exponential equation is by using logarithms. Another method is guess and check: guess values of t until we get an answer very close to 2. It is a good idea to keep track of guesses and answers, and make big jumps first, then smaller jumps later (see the table below).

   Therefore, it takes 12.9 years for the population to double. That's for *any* doubling, not just from 500 to 1000.

   Since we have the doubling time $D = 12.9$, we can write the equation in terms of base 2:

   $$P(t) = 500(2)^{\frac{t}{12.9}}$$

   We now have two equvalent functions for the small town population growth.

t	Ans.
5.0	1.30
10.0	1.71
15.0	2.23
13.0	2.01
12.5	1.95
12.8	1.98
12.9	2.00

2. The half-life of a radioactive element is 112 years. What is the annual decrease as a percent?

   Using the half-life equation with $H = 112$:

   $$M(t) = M_0 \left(\frac{1}{2}\right)^{\frac{t}{112}}$$

   The first step is to find the amount lost in one year:

   $$M(1) = M_0 \left(\frac{1}{2}\right)^{\frac{1}{112}} = 0.9938\, M_0$$

   This means that after one year 99.38% of the original mass is still present. To find the annual decay rate, we need r:

   $$r = 1 - 0.9938 = 0.0062$$

   Therefore, the radioactive material decays at 0.62% per year. Since we have r, we can write the equation in terms of the general base $1 - r$:

   $$M(t) = M_0\,(0.9938)^t$$

   Note: We did not need an initial value because the problem was solved in terms of *fractions* of the original amount. Question 1 can also be answered without the initial value; the LHS at the second step will always be 2, regardless of the actual population numbers.